Summary

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Principles	

State	The state of a quantum system is described by a non-zero vector in a Hilbert space.
Superposition	vectors describe the same states if they are equivalent in the projective Hilbert Space
Observable	A measurable quantity is described by a Hermitian operator on the Hilbert space.
Measurement	Eigenvalues of Hermitian operators are the measure outcomes.
Evolution	$i\hbar\lvert\dot{\phi}\rangle = \hat{H}\lvert\phi\rangle$
Collapse Postulate	After the measurement, the state will be projected on the eigenvectors.

Operators	

Uncertainty Principle	Minimal Uncertainty at gaussian package Uncertainty Principle is completely a consequence of statistic, as the trade-off between the spatial space and dual function space
Commutator	Commute operators share same eigenvectors $[\hat{q},\hat{p}] = i\hbar\hat{I}$ **Complete set of Commuting Observables (C.S.C.O**) means non-degenerated diagonalizable observables
Hermitian	Eigenvalues are real Eigenvectors of a Hermitian operator belonging to different eigenvalues are orthogonal Expectation of Hermitian operators are Real All expectation of an operator are real -> operator is Hermitian The eigenvectors of a Hermitian operator form a complete orthogonal basis set for the Hilbert space the operator acts upon.
Hadamard Lemma	$e^{sX}Ye^{-sY} = Y + s[X,Y] + \frac{s^2}{2!}[X,[X,Y]] + \frac{s^3}{3!}[X,[X,[X,Y]]] + \cdots$
Riesz Theorem	**Any Hilbert space is (anti-)isomorphic to its dual space. There exists 1 to 1 bijection between linear functionals F and vectors f . Such that we can make pairings $\langle F,f\rangle$.**

1D systems	

Piecewise Potential	$\phi'(0^+) - \phi'(0^-) = -\frac{2m}{\hbar^2}\int_{-\varepsilon}^{\varepsilon} dx(E-U)\phi$
Harmonic Oscillator	$Q, P\ dimensionless\ operator$ Ladder Operator to describe the relationships between discrete states $H = \hbar\omega\left(a^+a + \frac{1}{2}\right); [a,a^+] = I$

Quantum Dynamics	

Free Particle	$\phi_E(x) = Ae^{ikx} + B\mathrm{e}^{-\mathrm{i}kx}$ $k = \frac{\sqrt{2m(E-V_0)}}{\hbar}$
Gaussian wave package	$\psi(x) = N\exp\left(-(\alpha x - q)^2 + \frac{\mathrm{i}}{\hbar}p(x-q) + \frac{\mathrm{i}}{\hbar}\gamma\right)$

		$N = \left(\frac{2\mathrm{Re}(\alpha)}{\pi}\right)^{1/4} \mathrm{e}^{\mathrm{Im}(\gamma)/\hbar}$ $\quad \Delta q = \sqrt{\frac{1}{4\mathrm{Re}(\alpha)}}$ $\langle \hat{q} \rangle = q$ $\quad \Delta p = \frac{\hbar \lvert\alpha\rvert}{\sqrt{\mathrm{Re}(\alpha)}}$ $\langle \hat{p} \rangle = p$ $\Delta q \Delta p = \frac{\hbar}{2} \frac{\lvert\alpha\rvert}{\mathrm{Re}(\alpha)} \geq \frac{\hbar}{2}.$
	Heisenberg equation	$\frac{d\langle A \rangle}{dt} = \frac{i}{\hbar} \langle [H, A] \rangle + \langle \frac{\partial A}{\partial t} \rangle$
	Ehrenfest theorem	**The dynamics of expectation evolve like the classical quantities** $\frac{d\langle q \rangle}{dt} = \frac{\langle p \rangle}{m}$ $\frac{d\langle p \rangle}{dt} = -\langle \frac{\partial V}{\partial q} \rangle$
Lie Group and Lie Algebra	Representation of the Lie group	$\hat{D}(x) = \hat{I} + \mathrm{i} \sum_{j=1}^{n} x_j \hat{T}_j + \cdots$
	Generator of the group	$\hat{T}_j = -\mathrm{i} \frac{\partial \hat{D}}{\partial x_j}\Big\rvert_{x=0}$
	Infinitesimal Generator	$\boldsymbol{\hat{D}(x) = \left(\hat{D}(\Delta x)\right)^m \approx \left(1 + \mathrm{i}\Delta x \hat{T}\right)^m = \left(1 + \mathrm{i}\frac{x}{m}\hat{T}\right)^m \xrightarrow[m\to\infty]{} \mathrm{e}^{\mathrm{i}x\hat{T}}}$
	Lie Algebra	Translation in Phase plane: $[\hat{q}, \hat{p}] = i\hbar\hat{I}$ Harmonic Oscillator: $[\hat{a}, \hat{a}^\dagger] = \hat{I}, \quad [\hat{N}, \hat{a}] = -\hat{a}, \quad [\hat{N}, \hat{a}^\dagger] = \hat{a}^\dagger$ Quantum angular momentum $SU(2)$: $[\hat{J}_j, \hat{J}_k] = \mathrm{i}\hbar \sum_l \epsilon_{jkl} \hat{J}_l$
	Evolution Operator	If the Hamiltonian is an element of a Lie algebra Magnus form: $\hat{U} = \mathrm{e}^{\mathrm{i}\sum_j \alpha_j \hat{K}_j}$ Wei Norman Form: $\hat{U} = \prod_j \mathrm{e}^{\mathrm{i}\beta_j \hat{K}_j}$
Quantum Phase Distribution	No quantum phase space distributions will fulfil all the requirements below	
	1. A classical phase-space density is real and non-negative, i.e., $\rho(q, p) \in \mathbb{R}\rho(q, p) \leq 0$ for all q, p.	
	2. A classical phase-space density is normalisable, i.e., the integral $\int \rho\,(q, p) \mathrm{d}p\, \mathrm{d}q$ over the whole phase space has to be finite.	
	3. Expectation values of functions \$A(p, q)\$ of position and momentum are given by phase-space integrals $\langle A \rangle = \int A\,(p, q) \rho(p, q) \mathrm{d}p\, \mathrm{d}q.$	
	4. The p-and q-distributions are given by the marginals $\rho_p(q) = \int \rho\,(p, q) \mathrm{d}q$ $\rho_q(p) = \int \rho\,(p, q) \mathrm{d}p$	