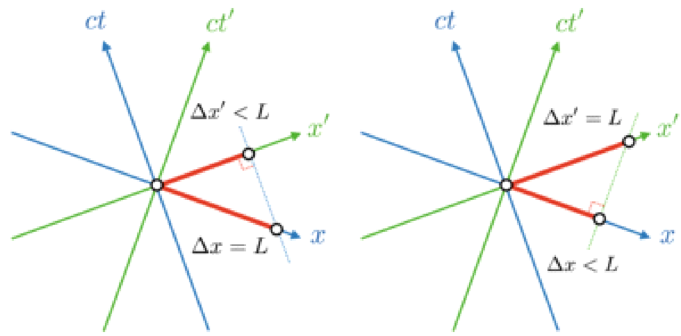
GR

Monday, May 30, 2022 9:22 PM

Special Relativity

x	The object is not an invariant. But an Object is a set of events in the spacetime diagram such that $O(x,t)$. Therefore, for different observers at a specific time, the object contains different events.  For ct, the object is located at his space. For ct', the object is located at his space. Therefore, an object is not a unique invariant set of events in the spacetime. Instead, the set of events of an object is highly related to the frame. Question: how can we recognize an object.

Lorentz Group

Lorentz transformation Λ	Proper $det\Lambda = 1$ Orthochronous $\Lambda^0_0 > 0$
Parity	$diag(1,-1,-1,-1)$
Time reversal	$diag(-1,1,1,1)$
PT	$diag(-1,-1,-1,-1)$
Poincare transformation	$x^{\mu'} = \Lambda^{\mu'}_{\nu} x^{\nu} + a^{\mu'}$

Metric

Definition:	$\boldsymbol{ds^2 = g_{ab}(x)dx^a dx^b}$
Polar	$ds^2 = dr^2 + r^2 d\theta^2$
Spherical	$ds^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$
Paraboloid	$ds^2 = (1+4\rho^2)d\rho^2 + \rho^2 d\theta^2$
Schwarzschild	$ds^2 = c^2\left(1-\frac{R}{r}\right)dt^2 - \frac{dr^2}{1-\frac{R}{r}} - r^2(d\theta^2 + \sin^2\theta d\phi^2)$
Rindler metric	$ds^2 = \alpha^2 x^2 dt^2 - dx^2$
Noether's theorem	$\frac{d}{ds}\left(g_{pd}\frac{dx^b}{ds}\right) = 0$ for g_{ab} doesn't depent on x^p

Covariant Derivative

Motivation: The derivative of (p,q) type tensor is not a tensor in general
$\nabla_c v^a = \partial_c v^a + \Gamma^a_{bc} v^b$ $\nabla_c v_b = \partial_c v_b - \Gamma^a_{bc} v_a$
$\nabla_a g_{bc} = \partial_a g_{bc} - \Gamma_{cba} - \Gamma_{bca} = 0$

Christoffel symbol Levi-Civita Connection	$\Gamma^a_{bc} = \frac{1}{2} g^{ad}(\partial_b g_{dc} + \partial_c g_{bd} - \partial_d g_{bc})$ $ds^2 = dr^2 + r^2 d\theta^2$

	$ds^2 = d\theta^2 + \sin^2\theta\, d\phi^2$

Parallel Transport

Motivation	It doesn't make sense to add two tangent vectors based on different points
Linear parallel transport	$P(x_A, x_B)^a_b = \delta^a_b - \Gamma^a_{bc}\delta x^c = \delta^a_b - \Gamma^a_{bc}(x^c_A - x^c_B)$
Equation of parallel transportation	$\frac{dv^a}{d\lambda} + \Gamma^a_{bc}\frac{dx^c}{d\lambda}v^b = 0$, the curve C is parameterized by λ $\frac{dv_b}{d\lambda} - \Gamma^a_{bc}v_a\frac{dx^c}{d\lambda} = 0$
Desire:	**metric connection**: $\boldsymbol{\nabla_c g_{ab} = \partial_c g_{ab} - \Gamma_{bac} - \Gamma_{abc} = 0}$ **Torsion free**: $\boldsymbol{\Gamma^a_{bc} = \Gamma^a_{cb}}$

Geodesic

Motivation	**The straight line, locally shortest**
Geodesic equation	$\frac{d^2x^a}{ds^2} + \Gamma^a_{bc}\frac{dx^b}{ds}\frac{dx^c}{ds} = 0$
	$\frac{v_b(x)dx^b}{ds} = const, if\ \nabla_a v_b + \nabla_b v_a = 0$

Riemann Curvature

Riemann Tensor	$R^a_{bcd} = \partial_c\Gamma^a_{bd} - \partial_d\Gamma^a_{bc} + \Gamma^a_{ec}\Gamma^e_{bd} - \Gamma^a_{ed}\Gamma^e_{bc}$
Equation of geodesic deviation	$\frac{D^2w^a}{ds^2} = -R^a_{bcd}u^b w^c u^d$
	$[\nabla_c, \nabla_d]v^a = R^a_{bcd}v^b$
Parallel transport around a rectangle	
Bianchi Identity	$\nabla_e R^a_{bcd} + \nabla_c R^a_{bde} + \nabla_d R^a_{bec} = 0$
Ricci Tensor	$R_{bc} = -R^a_{bca} = R^a_{bac}$
Scalar curvature	$R = g^{ab}R_{ab}$

GR

Energy Momentum tensor

$\partial_\nu T^{\mu\nu} = 0$
Symmetric
$T^{00} = u, T^{0i} = \frac{S_i}{c}, T^{i0} = cg_i$
S flux of energy, g momentum density, T^{ij} flux of momentum, u energy density

Schwarzschild Black Hole

metric	$ds^2 = c^2\left(1 - \frac{R}{r}\right)dt^2 - \frac{dr^2}{1 - \frac{R}{r}} - r^2(d\theta^2 + \sin^2\theta d\phi^2)$
S'child radius	$R = \frac{2GM}{c^2}$
Geodesic equation	$\ddot{r} + \frac{c^2R}{2r^2}\left(1 - \frac{R}{r}\right)\dot{t}^2 - \left(1 - \frac{R}{r}\right)^{-1}\frac{R}{2r^2}\dot{r}^2 - r\left(1 - \frac{R}{r}\right)\left(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2\right) = 0$ $ck^2 = g_{t\beta}\frac{dx^\beta}{ds} = c^2\left(1 - \frac{R}{r}\right)\frac{dt}{ds}$ $h = r^2\sin^2\theta\frac{d\phi}{ds}$
Connections	$\Gamma^r_{tt} = \frac{c^2R}{2r^2}\left(1 - \frac{R}{r}\right), \quad \Gamma^r_{rr} = -\left(1 - \frac{R}{r}\right)^{-1}\frac{R}{2r^2}$ $\Gamma^r_{\theta\theta} = -r\left(1 - \frac{R}{r}\right), \quad \Gamma^r_{\phi\phi} = -r\left(1 - \frac{R}{r}\right)\sin^2\theta.$

Kruskal

	Coordinates	
Interesting Scenarios	Deflection of Light	
	Fall into Black Hole	
	Newtonian Gravity]	
	Tidal Force	